Better Hedge Ratios

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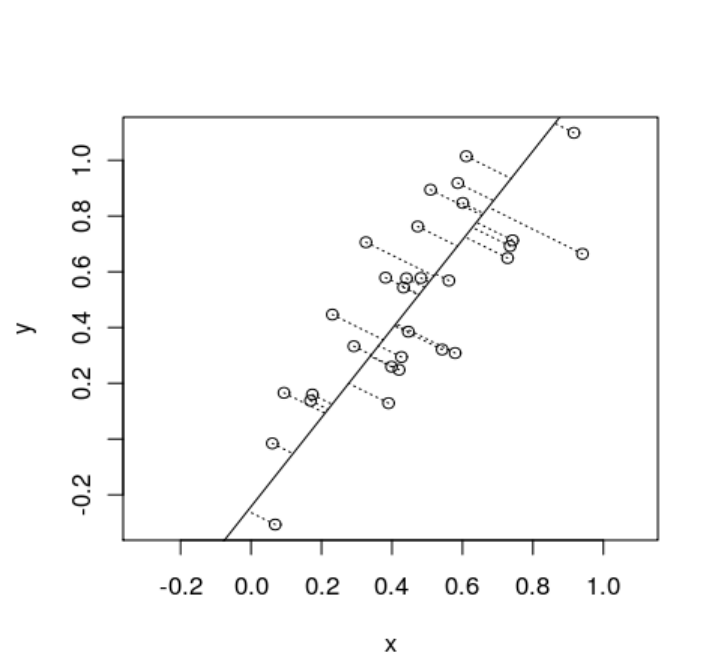


What is a hedge ratio?

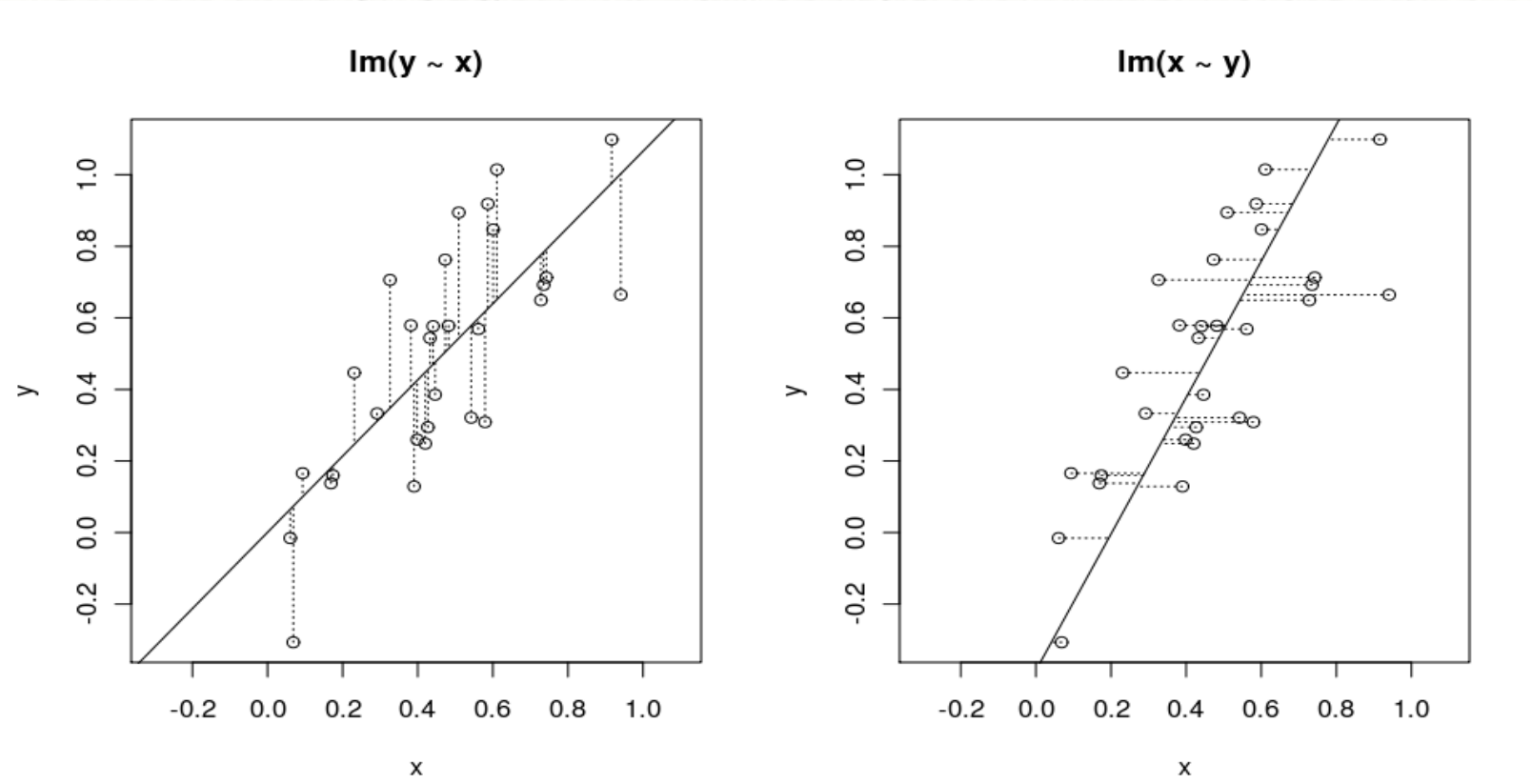
- Suppose we are trading a spread: long *Y* and short *X*.
- For each unit of *Y*, how many units of *X* should we sell short?
- That number is the *hedge ratio*.
- We calculate the spread as $S_t = Y_t - \beta X_t$ where β is the hedge ratio.

Hedge ratios are often calculated using ordinary least squares (OLS)

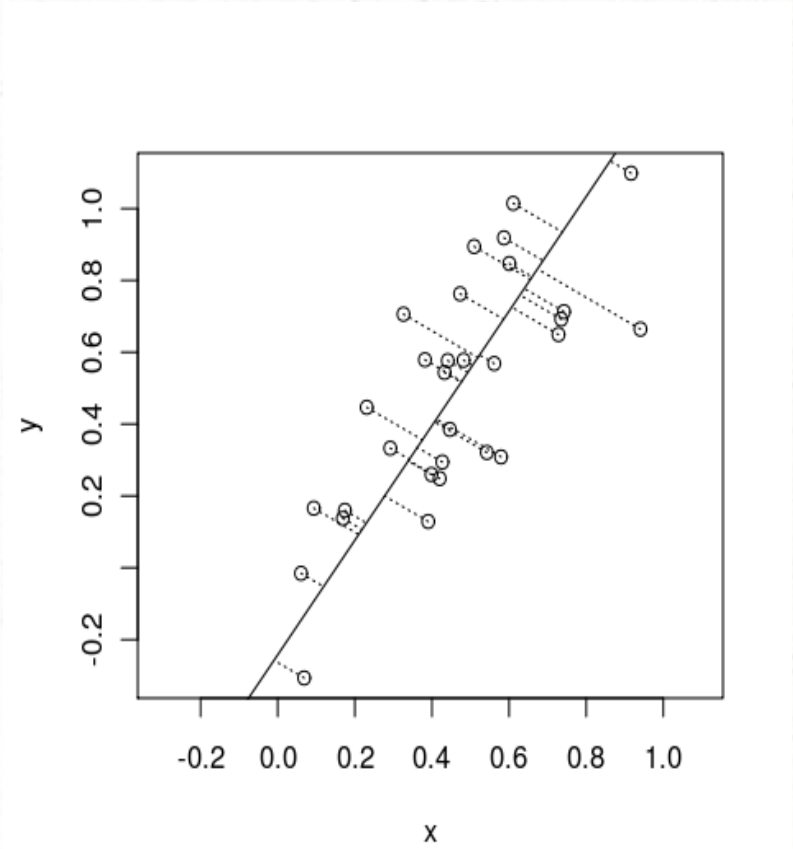
- Restate *X*, *Y* relationship: $Y_t = \alpha + \beta X_t + \varepsilon_t$.
- So spread is $S_t = Y_t - \beta X_t = \alpha + \varepsilon_t$.
- Hedge ratio is the β coefficient from linear regression. In R:

```
m <- lm(y ~ x)
beta <- coef(m)[2]
```

OLS is asymmetric: switching role of X, Y gives inconsistent H.R.



Total Least Squares calculates a hedge ratio which is symmetric.



- TLS minimizes orthogonal distance to regression line.
- Treats *X*, *Y* symmetrically: both are sources of variance.

TLS is easily calculated in R using principal components analysis.

- TLS for 2-asset spread:

```
r <- princomp( ~ x + y)
slope <- r$loadings[2,1] / r$loadings[1,1]
intercept <- r$center[2] – slope*r$center[1]
```

- Generalized TLS can handle more than 2 assets for multi-leg spreads.
- Can implement zero-intercept regression, too.

TLS: It's cool

- TLS treats the sides of the spread symmetrically, giving a consistent hedge ratio.
- Easily computed in R.
- Reference: *The Total Least Squares Problem*, Van Huffel and Vandewalle (SIAM, 1991)
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